Department of Mathematics

Course Profile

Course Number: MATH 324	**Course Title: Methods of Applied Mathematics I**
Required / Elective: Elective	**Prerequisite:** None
Catalog Description: Application of special functions, orthogonal series, boundary-value problems in mechanics and engineering, introduction to Sturm-Liouville systems. Solution techniques for boundary-value problems in curvilinear coordinates, integral transforms; Green's functions, potentials, applications.	**Textbook / Required Material:** "Sturm-Liouville Theory and its Applications", by M.A. Al-Gwaiz, Published by Springer, 2008. "Fourier Series and Boundary Value Problems", by J.W. Brown and R.V. Churchill, Published by McGraw-Hill Companiees, Inc., 2001.

Course Structure / Schedule: (3+0+0) 3 / 6 ECTS

Extended Description:

The goal of this course is to study various methods used for solving boundary value problems. The topics include: Sturm-Liouville problems, Orthogonal functions, Bessel's functions, Legendre's functions, Green's function techniques and Fourier integral transform. Most of the material to be covered can be found in chapters 1-6 of the first textbook and in chapters 6-9 of the second textbook.

The first chapter is about Sturm-Liouville problems (two-point boundary value problems, regular Sturm-Liouville problems, self-adjoint problems, inner products and orthogonality of functions, eigenvalues and eigenfunctions, eigenfunction expansions and mean convergence, completeness and Parseval's equality, nonhomogeneous Sturm-Liouville problems, singuler Sturm-Liouville problems). The second chapter considers Bessel functions (Vibrations of a circular elastic membrane (symmetric and general cases), Bessel's equation and Bessel functions, some properties of Bessel functions). The third chapter is devoted to Legendre polynomials (Dirichlet problem in a ball (symmetric case), Legendre's equation and Legendre polynomials, some properties of Legendre polynomials, Fourier-Legendre series, associated Legendre functions). The fourth chapter is about Fourier integrals (distributions, Fourier transforms, properties of the Fourier transform, convolution). The fifth chapter is devoted to Green's function (source functions, Green's function for ordinary differential equations, modified Green's function, Green's function for Laplace's equation).

Design content: None.	**Computer usage:** No computer usage required

Course Outcomes: By the end of the course the students should be able to:

1. derive a two-point boundary value problem starting from the heat conduction equation and understanding the physical meaning of this derivation **[2,3,6],**
2. classify a Sturm-Liouville problem as regular or singular **[3,6],**
3. know the basic properties of eigenvalues and eigenfunctions of a regular Sturm-Liouville problem **[3,6],**
4. find the coefficients in the eigenfunction expansion of a given function **[3,6],**

5. know the notion of mean convergence **[3,6],**
6. solve the nonhomogeneous Sturm-Liouville problems **[3,6],**
7. solve a sample problem of singular Sturm-Liouville problems (Chebyshev polynomials) **[3,6],**
8. derive the Bessel differential equation starting from the problem of the vibrations of a circular elastic membrane **[2,3,6],**
9. know integral forms and orthogonality properties of Bessel functions **[3,6],**
10. know the Gamma function **[3,6],**
11. derive the Legendre differential equation starting from the Dirichlet problem in a ball **[2,3,6],**
12. know orthogonality properties of Legendre polynomials **[3,6],**
13. know associated Legendre polynomials **[3,6],**
14. know the definition of the Fourier transform and understand convolution **[3,6],**
15. compute and apply Fourier transforms and use them to solve partial diffrential equations **[3,6],**
16. understand the concept of a Green's function **[3,6],**
17. conctruct Green functions for nonordinary diffrential equations **[3,6],**
18. conctruct Green's function for Laplace's equation **[3,6].**

[2] demonstrate knowledge of mathematics and mechanics to construct, analyze and interpret real world problems,
[3] demonstrate the ability to apply mathematics to the solutions of problems,

[6] have a basic knowledge of the main fields of mathematics and mechanics, including differential equations, elasticity theory, fluid mechanics,

Recommended reading:

"Elementary Differential Equations and Boundary Value Problems" (Chapter 11), by W.E. Boyce and R.C. DiPrima, Published by John Wiley & Sons, Inc., 1992.

"First Course in Partial Differential Equations with Complex Variables and Transform Methods" (Chapters 5,7,10), by H.F. Weinberger, Published by Dover Publications, Inc., 1995.

"Partial Differential Equations-An Introduction" (Chapters 7,10,12), by W.A. Strauss, Published by John Wiley & Sons, Inc., 1992.

Teaching methods: Preparatory-readings, lectures, discussions, assignments

Assessment methods:Midterm exams, final exam

Student workload:

Preparatory reading ....42 hrs

Lectures ……………………………..42 hrs

Assignments ……………………… ..43 hrs

Discussions …………………………14 hrs Midterm exams……………………… 6 hrs Final exam ………...………………… 3 hrs **TOTAL ……………………………… 150 hrs … to match 25 x 6 ECTS**	
Prepared by : Husnu A. Erbay	Revision Date: 08.02.2010